# Spatial lines

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PROLOGUE

## A void that can be filled

Roberto Doberti

The phrase that stated "nature abhorres void" and the Latin expression that follows: *horror vacui*, have always intrigued me. For centuries they were more than language expressions, they were strong convictions. As nature could unexpectedly "abhor" or become horrified it was necessary to spare her these bad moments. So, everything was filled with ubiquitous ether, which temporarily pacified those spirits that needed to feel complete, fulfilled, satiated (as can be seen, the correspondence with nutritional metaphors are not irrelevant).

Once this illusion was destroyed, we seem to, but only seem to content with a nature that is "almost void". On any scale —from cosmos to atoms —, it appears that what separates (what is not) greatly surpasses what is materialized (the correspondence between the wealth of powerful people and the scarcity of excluded people is also relevant here).

I suppose this is not a legitimate argument to mention my general disagreement with what is full, due to the lightness with which entities are incorporated and because the approaching and presumed saturation is dreamt of; and with what is almost void, for the difficulty in thinking new and multiple existences, imagining a rarefied field, where drought and shortage rule.

Many years ago we determined five Types of Figures in our System of Figures. The Types were defined according to the dimensions of the space in which the shapes were included and to the dimensions of each figure in relation to its bounding space. One of those Types is constituted by those shapes included in a bounding space of three dimensions which have two dimensions less than such space. So, we defined Spatial Lines.

From the beginning, this place seemed very interesting and able to hold entities of particular beauty and structuring values, in short, a promising realm.

We knew this place was not void because we already knew the helix, the lines that modern furniture design had already built with curved tubes and a few more. It is also truth that these entities only showed us a horizon of splendor which was both, longed for and elusive. As Adam would have said about Eden: “Everything is very nice, but there is not much social life”.

So, I celebrate with great enthusiasm this book: “Spatial Lines”.

Thinking it thoroughly, there are many reasons for this celebration. To begin with, the longed-for and multiple entities that simultaneously speak of their harmony and precision that populated this field. Social life flourishes among the Spatial Lines.

It is not less important that this book is the product of diverse hands, none of which loses, due to this diversity, their singular tenderness in the caress of these forms.

Just like these hands are diverse and convergent, so the disciplines associated to these births or revivals are. General morphology, mathematics, industrial and graphic design, fine arts and didactics converge in concert. The moment arrives, and it is the best moment, in which you no longer know where one or other discipline begins or, better saying, if one of them is not present any more. Only the signification level can produce these miracles: but in Morphology miracles exist (or could it be that Morphology is a miracle?)

There is also an overlapping and recurrences of time. To recover the ancient Greek (Archimedes, Menaechmus and in particular Archytas), to place them in the same space with Moebius, with contemporary designers and artists and with students of the FADU is a task that requires courage and enough lack of shyness.

To conclude, I should point out that we have the history or the legend that rose with Archimedes death. He was killed during the capture of Syracuse by a soldier who could not understand the perception of time of the wise man and much less his set of values.

Archimedes was buried in a tomb, which according to the instructions of the geometer, should carry as gravestone or monument the image of cylinder circumscribing a sphere. The tomb was lost until Cicero remembered the fact and recovered it. The truth is that this posthumous triumph of Archimedes was not definitive, because this monument is no longer known.

This book can be understood as this well deserved recovery. There is no identification with the stones of the past, but there is a bond with the spirit of research and qualification of the way of looking and thinking that Archimedes would surely have appreciated.

*May, 2010*

PROLOGUE

## Vitality in relentless production: from geometry to product

Claudio Guerri

It is well known Edmund Husserl's worry, in his last years, about the crisis of Western Culture. It is well known too, his belief that humanity's withdrawal of the world of life-world would be one of the main causes of such crisis, and he thoroughly looked for ways of solving this problem that distressed him.

When time came to write the prologue of the book *Spatial Lines* —which can enroll in Husserlian tradition- , the confrontation from a phenomenological point of view with empirical operating capacity required of "geometrical shapes" in industrial design practice, allows this short reflection, on the light of one of the text that the philosopher produced, beset by physical illness and the foreseen effects of the Nazi crisis. In his manuscript from 1936, which he wrote for his own use, entitled after his death: "The question about the origin of Geometry as an intentional-historical problem", Husserl considers his reflection on geometry as a possible proposal tending to suture the divorce between everyday life and science.

In the text, Husserl gathers under the name of Geometry all disciplines that deal with "form", in which mathematical existence moves in phenomenological time-space. When he inquired about the origin, his intention was to go back, in the most original sense —according to which, geometry was born one day and since then it is present as a millenary tradition and is preserved for us-, in the living "relentless production", but as the same time as "tradition". He sustains that this tradition is conceived by "human activity". Humanity, to which the first creations that emerged of the available, raw and "informed by the spirit" materials; gave form to novelty. Even if today we prefer to speak of a correspondence of different kind of values to those materials, consequence of experience and cognition, the time metaphor does not lose its validity.

Evidently, Husserl follows, geometry should have been born from a first conceptual acquisition of "primary creative activities" and it is necessary to understand its "persistent" way of being: not only it is a forward movement that progresses without stopping from one conceptual acquisition to the other, but a continuous synthesis in which all the acquisitions keep their value, becoming together an acquisition, so that each acquisition will be premise of the following stage.

The former paraphrase locates us in the core of this thought: Husserl did not consider Geometry a merely theoretical question, as the already mentioned "primary creativity" could be but- by the middle of the desolate twentieth century- he preferred to focus on the social operating capacity of this theoretical practice. Science, and specifically geometry, should have had, in a historical beginning, their origin in a "productive act" logically presented, at first with a "project" conformation —an idea, a geometrical diagram-, where some hypothesis is sustained, that then will be materialized in an "event " that we can, as today readers, assign to the morphological or design area.

Going further: geometrical existence does not have physical existence but it occurs in the morphological field; geometry has an existence of "being there", objective, for everyone. It is an ideal objectiveness, where a "logical activity" has to be considered, specifically connected to verbal language. This activity that deals with both, logic and project, definitively is characteristic of a class of conceptual products of the culture to which they belong, not only the conceptualizations and formal scientific productions, and sciences themselves, but also, for instance, the formal productions of literature, works of architecture and products of design.

This book –an indispensable link in the geometric and morphological knowledge applied in industrial design-, presents us the actual state of conceptualization of ideal objects contingent to this science; contingency that includes systems and software of visual representation that enable the development of the semiotic chain: ideation, representation/production and use. In fact, representational technologies constitute the conditions of possibility, supportive and concrete so that certain ideal objectivities could finally materialize as design objects: the intentional reactivation of a significative value should forcefully precede and condition the empirical determination of an object.

As can easily be understood, the "material thing" implies some kind of *"res extensa"*, and geometry, morphology and the ontological and operational disciplines that enable the eidetic moment of the structure of the "thing" too: the spatial form. At the same time both, conceptual geometry and operative morphology are responsible of the naturalization of physical space in terms of representational modes.

The diverse production of articles of this book gives plenty of examples that, in Husserl's way, while covering geometric milestones join temporal extremes transforming them in spatial proximities. The proposal of the book and of its compilator, Patricia Muñoz, fills a space where vitality of relentless production" unites, within the scope "of tradition"...We have only a wish left: to expect that its readers will get the same benefit and pleasure I have had reading it.

May, 2010

## Relevant explanations

This book has many authors. Some belong to a research group in the IEHU, Laboratory of Morphology of the FADU, University of Buenos Aires that fortunately harbors and promotes our projects. Others belong to a teaching group that transferred this topic to the courses of Industrial Design undergraduate program, FADU, UBA. Some of us belong to both groups, and we also have three special guests, that I want to thank especially for their contribution.

To Nina Enrich, who made comprehensible for non mathematicians the demonstration of Archytas solution for doubling the cube, patiently enduring my doubts and questions.

To William Huff who, long time before we started our research, explored with unmistakable orientation the world of nonorientable surfaces, which meet spatial lines in their edges and materializations. The experience of timing makes evident the potential of looking intentionally to a form, enabling interesting alternative visual interpretations of the same material configuration.

To Robert Wiggs, who dissolved the limits between polyhedra and spatial lines with his twisted loops, showing his marvelous sensibility in his sculptures whose origin can be found in spatial lines combined with an enormous capability to imagine and design in space.

I have two thank specially two persons who generously accepted to prologue this book. However, my gratitude goes beyond this. To Claudio Guerri, who introduced me to William Huff and Robert Wiggs, who were relevant instigators of this work. To Roberto Doberti, for raising our curiosity, giving a name and properties to these exciting shapes through his work, the “Sistema de figuras”, providing a frame of reference for their study.

### Editorial structure

The journey suggested in this book has three moments. The first one, antecedents, includes the origin of this Project, which was totally accidental (if we accept that fate exists). It also comprises the first steps we made to introduce ourselves in the topic of spatial lines and the thoughts of those people who helped us to understand the topic. Their fascinating explorations allowed us to foresee that the road ahead was very interesting.

In the second one, the research, graphic and industrial design products were analyzed making evident the relevance spatial lines in design practice and the need of conceptual definitions that were produced throughout this investigation.

In the third one, educational investigations, the findings of the research project were transferred to the courses of Morphology for Industrial Design in order to check and extend this knowledge.

Finally, the Appendix includes some of the inquiries in intersections between shapes and the variations we carried out, which became a central point in the development of this subject-matter.

We hope you enjoy it.

The editor

**PART 1.** Antecedents

CHAPTER 1

## Oblivion and recovery of forms[1]

Patricia Muñoz and Juan López Coronel

New technologies provide innovative resources to understand forms as they help to visualize, generate and materialize our projects. They also contribute in the rescue of forgotten knowledge.

Internet is a diverse registry of human knowledge. Even if its supposed distribution in the world population is misleading—26% of penetration is the data of December, 2009 (Internet World Stats) - it is an unquestionable media for connecting researchers on the most diverse areas.

The amount of information available is a disadvantage for its users, even if it is a benefit to avoid control. For example, a search engine can provide 52.000.000 items on the cube, in different languages. Unapproachable. Fortunately, search engines can filter information making it accessible, providing pleasant surprises.

### Unexpected encounters: Lemniscates in space

The origin of our research in the topic that is the object this book can be found in an unexpected encounter. We casually found a spatial line, Archytas curve, in the site *Encyclopédie des formes mathématiques remarquables* developed by Robert Ferréol. This shape is the result of the intersection of horn torus with a cylinder of the same diameter of its generative circle, which is tangent to its exterior surface to its axis, as shown in Figure 1.

Figure 1. Archytas curve as boundary curve of the intersection between a cylinder and a torus.

Progressing in this inquiry we found out that these lines had been studied by the ancient Greeks, beginning with Archytas of Tarento (430-350 BC), Pythagorean mathematician, statesman and philosopher. He was the first one to solve one of the well known mathematical problems in antiquity: the duplication of the cube. [3]. "Eratosthenes (300 AC) reports that the inhabitants of the Greek island of Delos were beset by a plague and, when they consulted an oracle for advice, were told that, if they doubled the size of a certain altar, which had the form of a cube, the plague would stop." (Huffman, 2008) The problem was to find the size of the side of the cube which would double the volume of the original altar. After unsuccessful efforts the Delians presented the problem to Plato in the Academy. Archytas was the first to reach a solution, which was remarkable because solved the problem through a construction in three dimensions. This way of thinking was uncommon in his time. He used the intersection line of the previously mentioned surfaces, describing the movement of their generative lines. Plato criticized this solution because in his vision, the value of geometry resides in its ability to turn from the sensible to the intelligible domain. In his opinion Archytas "was not focusing on the intelligible world but on the physical world and hence destroying the value of geometry." (Huffman, 2008)

In spite of the temporal distance, the relation between the physical and intellectual world is a relevant issue in the study of forms for industrial design. What we design in the virtual condition of our drawings and models –no matter in which substance: paper or bits- will be eventually manufactured in more or less numerous series. The feasibility of our projects considerably relies in the designer's rigorous knowledge of its form.

Following the progress in the knowledge of spatial lines we find the work of Eudoxus of Cnidus (408-355BC). He studied with Archytas, was astronomer, mathematician and physic. He developed a planetary theory, which consisted in a number of concentric rotating spheres of the same diameter, rotating about axes which were oblique to one another, in different directions. So he came to a dynamic definition of another spatial line belonging to this family [4], another lemniscate in space, understanding it as the trajectory of a point on the equator of one of these spheres. (O'Connor, J. J. et al)

Its diagram can be seen in Figure 2.

Figure 2. Sketch of the two spheres rotating with oblique axis in the determination of Eudoxus curve

We obtain an easier explanation of this curve, although static, if we define it as the intersection of a cylinder with a sphere, tangents in their exterior, as can be appreciated in Figure 3.

Figure 3. Curve studied by Eudoxus of Cnidus, intersection of a sphere and a cylinder.

Another line, of the same family, was the curve of Viviani. Vincenzo Viviani (1622-1703), disciple of Galileo, created this curve looking for an answer to a mathematical problem he raised related to architecture: he asked how four equal windows could be cut on a hemispherical dome so that the remaining surface can be exactly squared. (O'Connor, J. J. et al)

Figure 4. Curve of Viviani as an intersection of a sphere, a cylinder, a cone and a parabolic cylinder

Other curves that belong to the same family are the bicylindrical curves, obtained by the intersection of two cylinders of different diameter, tangent according to the image shown in figure 5. There is not much historical information of the study of these lines, compared to the ones previously shown.

Figure 5. Bicylindrical curve

## Morphological encounters in different time and space

Viviani´s curve appeared in a research project we were developing on surfaces created by double rotation of the generative line. We discovered it was the edge of some of the shapes we produced in that exploration.

Figure 6. Surface of double rotation, relation to the sphere and Viviani´s curve.

In Figure 6, we can observe a surface created by a semi-circumference that goes through a double rotation, with intersecting axis; touching the generative line in one of its endpoints. We can also see its relation to the sphere and to Viviani´s curve.

This opened new roads of inquiry and also the possibility of having a better understanding of the shapes we had produced in this way. In Figure 7 we can observe a fragment of de surface depicted in Figure 6, using a quarter of circumference as generative line. Its edge is also inscribed in the surface of a sphere and can be determined by the intersection of this sphere with a right cone whose generative line is at 45 degrees from the axis.

Figure 7. Fragment of the surface of Figure 6. Relation to the sphere. Determination of its edge through the intersection of a sphere and a circular straight cone whose vertex lies in the spherical surface.

It is remarkable that a double circular straight cone is the one that defines the edge of the complete surface of double rotation, the curve of Viviani, as its vertex is placed in the surface of the sphere. This relation can be seen in Figure 8. The conic surfaces, that determine both edges, are placed on a line that has the same angle as its generative line, 45º.

Figure 8. Relations between the intersections of cones and spheres and the determination of the edges of both surfaces.

Finally, we would like to notice another interesting encounter. We found another curve belonging to this family in an industrial design product, the Ripple Chair, designed by Ron Arad. By means of a systematic constitution, through the extension of the generative line, he obtains the seating surface. It is remarkable that the materialization shows the line and allows the visualization of the intersections between the different lines.

Figure 16. Ripple chair designed by Ron Arad

## Resources and contributions

After this research, we still have some questions unanswered. Why are some shapes forgotten? Could it be because at the moment they were not useful or desirable, or could it be because there were no elements to draw or materialize them? Today we enjoy impressive conceptual and operative instruments, unthinkable for those first researchers. They increase our competence but they also carry new obligations. If someone studied these shapes with minimal resources, should it be our responsibility to expand this knowledge from our time, bountiful of means?

We understand that digital technologies are specifically useful in the *exploration* of these complex forms because they favor a fast visualization and they also include the visual analysis of its curvature. Thinking spatial lines as intersection of surfaces allows us to control their shape, changing the parameters of this operation and of the figures involved in it.

In figure 9, a series of bicylindrical lines are illustrated, which change their configuration and proportion while modifying the diameter of one of the cylinders. This was the starting point of various explorations that are shown in Chapter 8.

Figure 9. Changes in one of the cylinders that determine the bicylindrical curves.

These images can be easily obtained through CAD, allowing us to use spatial lines —

created by intersections- as structuring lines in the *generation* of new surfaces.

For example, Figure 10 shows a surface created from an asymmetrical spatial lemniscates. Even if this surface seems complex, it is developable. Its planar net can be obtained with one of the CAD tools, allowing the rapid construction of 3D models to check their design.

Figure 10. Developable surface generated from an asymmetric spatial lemniscate.

The visualization, analysis and handling capability of designers is enhanced through digital modeling. It allows us to create surfaces through non-uniform, complex transformations as the ones exposed in the next chapter.

CAD-CAM software contributes to the materialization of our projects. In Figure 15 we can see images of handmade models of the surface of Figure 6. Even if they were useful during the research, they do not reflect accurately the shape as the model in Figure 16 does, which was produced by CNC milling.

Figure 15. Handmade models, with wire and “cartapesta”, of the surface of double rotation of figure 6.

Figure 16. CNC milled model from the CAD drawing of Juan López Coronel

## Some conclusions and deviations

We consider that our projecting possibilities can be expanded with the recovery and reinterpretation of the relegated knowledge we have related. The exploration and the analysis of this forgotten knowledge, with the use of digital media, create new opportunities of both conceptual inquiry and concrete realizations of our projects.

This chapter is just a sample of some of the roads we traveled in this encounter of morphological knowledge, originated if different periods of time. Rediscovering the passion of ancient researchers on these forgotten shapes we will be able to recover them, creatively using the conceptual and operative instruments to extend its use and to transform them, making use of its morphogenerative potentiality. So they will be able to re-enter to our dreamt and built habitat, restructured through design.

## Notes

[1] This work is a revised edition of a paper presented at the *V Congreso Nacional de SEMA, Sociedad de Estudios Morfológicos de la Argentina*, Resistencia, Chaco, 2005

[2] According to a classification of the “Sistema de Figuras” (System of Figures), Doberti et al. (1971). In this proposed order spatial lines are defined as “figures that can be included in a space of not less than three dimensions, and that have two dimensions less than this space”.

[3] The first answer was to build a second altar that duplicated the volume but was not a cube any longer. The second attempt was to build a new altar with its size twice the side of the original altar, failing because the volume is increased eight times. In Chapter 3 there is a more profound and extended explanation of Archytas solution.

[4] A dynamic and simpler definition can be found in the site of Ferréol, R. (1993), in the Hippopede curve.

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CHAPTER 2

## Transformations: Generation and materialization of a surface

Juan López Coronel

The drawings show the work which started with the analysis of the spatial line that was studied by the Pythagorean philosopher and mathematician Archytas of Tarentum.

This curve is the result of the intersection of two surfaces, one generated by rotation (torus) and the other by translation (cylinder) that use the same circumference as generative line.

By joining two sectors of this intersection it is possible to create a new spatial generative line that is continuous and closed. This 3D curve allows us to rebuild both primitive surfaces.

(Figure 1)

(Figure 2)

Using this generative line, two spatial surfaces were created. The first one was a surface of rotation with homogeneous scaling of the generatrix. The second one used a directional scaling.

(Figure 3)

(Figure 4)

A mould was built with CNC milling to produce a prototype of the surface. The final object simulates the texture and appearance of rocks. This feature was obtained after several trials using different combinations of materials.

(Figure 5)

(Figure 6)

The proportions of the primitive surfaces were modified. The inner diameter of the torus was increased, establishing a relation 1:3 to the generative line, in order to provide a structure with a central area which was more suitable and productive.

(Figure 7)

One of the materializations designed for this surface exhibits the fundamental and constitutive scaling of the generative lines. However, it is possible to heighten and make evident other spatial lines which can be found in the abstract surface.

CHAPTER 3

## Archytas and geometry

Rosa Nina Enrich

### Introduction

*No subject loses as much as*
*when it divorces from its history as Mathematics have*
*Bell [1]*

The analysis of historical processes in the development of mathematics can show the way in which methods, ideas, concepts and theories of this discipline arise, how they are systematized and developed. It can be extremely useful to explore the beginnings of a concept, the difficulties that mathematicians were confronted and the ideas that emerged in dealing with new situations, the problems they solved, the area in which they were applied, the methods and techniques developed, how they forged definitions, theorems and proofs, the thread between them to build theories, the physical or social phenomena they were explaining, the spatial and temporal context in which they appeared, how they evolved to its present state, how cultural issues were linked, the everyday needs they solved. In short, to know (in the Kantian sense) the way from intuitions to ideas, and then to the concepts. Although this chapter is focused on the analysis the work of Archytas as a mathematician of ancient Greece, it seems very important to give a historical context by briefly describing three classical problems of geometry that marked the lives of ancient mathematicians (some contemporary, others after Archytas). The quest for their solution generated many discussions among mathematicians throughout history.

They are: squaring the circle, the trisection of an angle and the doubling of the cube

The first was to find geometrically the measure of one side of a square depending on the extent of the radius of a circle so that its area is the same as that of the circle. It has been proved that this is unsolvable.

The second was to find the way to divide an angle into three equal parts using only an unmarked ruler and a compass. This task is generally impossible.

The third, was to find the measure of the edge of a cube so that its volume doubled that of another of known edge. Of all three problems this is the only one that arguably has a solution, but does not fulfill the requirement that it be buildable only with unmarked ruler and compass [1]. There are several solutions proposed in Classical Greece and the most prominent of them, as it was unusual for the time, is the one proposed by Archytas.

The three classical problems of Greek geometry had one thing in common: they could not be solved by unmarked ruler and compass and this was the great difficulty that determined the need to search for other means beyond those used so far. In fact, all three problems have no solution using unmarked ruler and compass.

The solutions involving marked instruments were called mechanical solutions [2]. Plato said that mechanical procedures irremediably lost the most sensitive part of Geometry; and we fully share his view. This is because it is necessary to solve a problem by means of a demonstration of its solution so that it can be said "This is the solution", because otherwise we would be talking only of verifications. We know that in the 5th Century (BC) Hippocrates of Chios made the first relevant contribution to the problems of squaring the circle and duplicating the cube. He also studied the problem of the trisection of an angle and although he found a direct way to do it, this way does not apply to any angle and therefore has no value as a general solution because it is not only particular but also mechanical.

Of the three problems we have mentioned, our interest here is on the third: the doubling of the cube; and we shall analyze particularly the work of Archytas rescued by Eutocius in the 2nd century (AD).

**Where history blends with legend about the birth of the problem**

It is said that the origin of the doubling of the cube arose in the mid 4th century (BC), when the citizens of Delos appealed to the oracle at Delphi to learn how to contain the plague invading their city. The oracle replied that they should double the altar of Apollo, which had the shape of a cube. This answer from the oracle is not surprising since there are already recorded problems about the size and shape of the altars in the early manifestations of Hindu literature, probably arrived in Greece from the hand of Pythagoras.

The fact that this problem had been addressed previously, may have its origin in the legend of King Minos who while visiting the construction of a tomb in cubic shape for his son Glaucus warns that it was small according to his expectations and says "Too small is the tomb you have marked out as the royal resting place. Let it be twice as large. Without spoiling the form, quickly double each side of the tomb". This idea contains an error, since doubling the sides of a cube results in a volume eight times the original, as it is seen in Figure 1. That's how this became a problem for mathematicians since then.

The legend of Minos would locate the statement of the problem in the period of splendor of the Minoan era, roughly arround the year 1600 BC.

Figure 1. The intuitive solution to double the edge of the cube shows how the volume is eight times the original.

From Ancient Greece, we must highlight three major attempts to solve this problem:

- The one by Hippocrates reported by Archimedes in his book *On the Sphere and cylinder.* It was based on the determination of two means proportional between a measure and another [4].He never got to formalize the solution.

- The one by Menaechmus, who not only discovered conics but also studied a number of their properties, at least enough to provide two simple solutions to the problem of Delos by means of the intersection of an hyperbola and a parabola.

- The one by Archytas, who found the two means proportional solving the problem with three surfaces of revolution.

If Hippocrates reduced the spatial problem of doubling the cube to a metric problem in the plane, Archytas took it back into space (we have not found in the literature any references of why Archytas chose this kind of approach to solve the problem).

They were followed by others, who tried to perfect the proposed solutions.

Although many methods were invented to duplicate the cube and many remarkable discoveries appeared during the attempt, the ancient Greeks would never find the solution that really wanted: *one that could be done with ruler and compass.* They could never find such a construction because it can't be done, as we will justify later. However, there was no way they could ever prove such a result, since it required mathematical concepts that went far beyond what they achieved. It is fair to say, however, that even though they could not prove that a ruler and compass construction was impossible, some of the best ancient Greek mathematicians foresaw that this was really impossible.

To understand the impossibility of the required solution we shall appeal to Modern Algebra, where the problem can be stated as follows: For the volume of a cube of edge "a" to be twice of that of another cube of edge "b" it must be verified that:

$$V_a = 2 \cdot V_b.$$

Since: $V_a = a^3$, y $V_b = b^3$,

Then: $a^3 = 2 \cdot b^3$

That is: $a = \sqrt[3]{2} \cdot b$.

The lack of a method to construct this cubic root with a ruler and compass is what makes impossible the solution they sought.

However, since the time of Hippocrates it was known that the solution of the problem was to find two proportional means between two quantities "a" and "b" so that the volumes of cubes of sides "a" and "b" were on a 2 to 1 relation.

Their aim was first to determine them and then demonstrate that it is possible construct them with ruler and compass.

Today, using modern algebra, we know that these proportional means are $\sqrt[3]{2}$ and $\sqrt[3]{4}$, and so the proposal made by Hippocrates and taken by Archytas in his quest to prove their existence would be expressed like this:

$$1 : \sqrt[3]{2} :: \sqrt[3]{2} : \sqrt[3]{4} :: \sqrt[3]{4} : 2,$$

which is the same as:

$$\frac{1}{\sqrt[3]{2}} = \frac{\sqrt[3]{2}}{\sqrt[3]{4}} = \frac{\sqrt[3]{4}}{2}$$

and the relationship between the volumes would be:

$$1 : 2 :: 2 : 4 :: 4 : 8$$

which is the same as:

$$\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$$

as shown in Figure 2.

Figure 2. Approximated construction of cubes of edges measuring 1, $\sqrt[3]{2}$, $\sqrt[3]{4}$ and 2, with volumes 1, 2, 4 and 8 respectively.

In the second half of the seventeenth century and early eighteenth centuries, many mathematicians were devoted to finding ways to double the cube. Among them were Descartes, Fermat, Huygens, Viviani and Newton.

Descartes considered not only the problem of finding two proportional means, but also came to consider four. Fermat went further and considered certain classes involving *n* proportional means. Viviani solved the problem with the help of a second order hyperbola. Huygens, in 1654, suggested three methods of solution. Finally, in 1707, Newton suggested several methods but chose one in which uses Pascal's snail (the cardioid is the best known of this family of curves).

This brief historical background to the problem allows us to understand the complexity of some mathematical problems which appear to have simple solutions.

Here is one of the keys to those who are enthusiastic about the scope of this discipline:

*In mathematical terms, intuition can be a good starting point. But do not forget that Mathematics is the discipline of demonstrations. Only that which can be demonstrated constitutes a step that allows raising up to the next.*

Highlighted this important issue, we devote ourselves to the analysis of Archytas' *solution* to the problem of doubling of the cube.

## Archytas and the doubling of the cube

### About Archytas

Archytas of Tarentum (about 428-350 BC) was a mathematician, philosopher and politician who developed his activities in the first half of the fourth century (BC), contemporary with Plato. He was one the last famous figures of the early era of the Pythagoreans. It was the first to identify Logic (closely associated with Arithmetics), Geometry, Astronomy and Music as the canonical sciences, which became known as Quadrivium in the Middle Ages. These sciences added to Zeno's Trivium (Grammar, Rhetoric and Dialectics) were the seven liberal arts. (Boyer, 1986)

We say this to stress the importance of his career, not well known outside the field of Mathematics. His research does not come to classrooms as frequently as those of other Greek mathematicians such as Pythagoras, Archimedes, etc. The problem is that very few of his original works, with an air of authenticity, have been recovered. Two remarkable of his works are: the reports of his solution to the doubling of the cube, resulting in the discovery of a significant curve known as Archytas' Curve (Ferreol, R.), and his work on musical harmony (Huffman, C.) , which marked an important contribution to the musicians of his time.

### About the proposal to the doubling of the cube

Archytas of Tarentum provided a solution using a geometric construction involving a cylinder, a torus and a cone of revolution (see Figure 4 b).

Remember that, as noted above when we told the story of Delos, the intuitive solution to doubling the cube was doubling its side. However, if this is done, you get a cube with a volume 8 times the volume of the first cube, as we noted. This allows us to understand that the side that doubles the volume of a cube of side one must have a measure between 1 and 2. Hence his work starts with the conviction that the solution to doubling the volume of the cube means finding two proportional means between b = 1 and a = 2 constructible with ruler and compass so that with the least of these measures could serve as the edge of a cube of volume equal to 2.One might ask: why two proportional means? For if we see that the volumes for the cubes of edges 1 and 2 are 1 and 8, then it is possible to find among them two proportional means that allow us to form the sequence 1, 2, 4, 8 which is a geometric progression in which 2 and 4 are the proportional means between 1 and 8. This is the case of the volumes. For the edges, the problem is reduced (is reduced?) to find two proportional means between 1 and 2, of which the smallest corresponds to the edge of the cube of volume 2. This is the desired solution.

The remarkable thing about Archytas's reasoning is that it relied on a construction made in the domain (then considered superior) of curved surfaces. The result is consistent with the discovery of the Pythagoreans, of Plato and Theaetetus of the construction of the five regular solids from the sphere. These were splendorous times for Geometry at the time.For the first time, there were reported three-dimensional constructions represented in the plane and they were given a tremendous value. This is perhaps the reason why Archytas sought a solution to the problem in this area.

We begin the study by describing the characteristics of the three surfaces of revolution involved. And we have emphasize that the basis of the solution is to assign to a diameter and to a rope of the same circumference measures whose ratio is 2:1. This is seen in Figures 3 and 4 where we have:

1) A torus generated by a circle of *AC diameter* of length 2, which revolves around a coplanar line and tangent to it in A. Therefore, the generated torus has zero internal diameter, this means no passing space.

2) A right cylinder of *diameter AC,* with its axis parallel to the axis of the torus and moved a *distance AC/2* from it. Note that the generating circle of the torus has the same diameter as the guideline of the cylinder and are in perpendicular planes.

**Figure 3.** Intersection of the torus and cylinder. We see part of Archytas' Curve, the result of the intersection between the two surfaces.

3) A section of right cone whose axis contains AC. Its generatrix is determined by the direction of the segment AB. This segment is a string of the circumference guideline of the cylinder and has *length 1.* The extension of the segment AB cuts in point D to the tangent to the circle ABC drawn at point C. The triangle ACD is determined so that as it is rotated around its side AC it generates the right cone sector involved in construction (Figures 4 a and 4 b).

**Figure 4 a.** Intersection of the torus and cylinder. Details on the generation of the cone.

**Figure 4 b.** Intersection of the torus, cylinder and cone.

The surface of the cone has 4 points of intersection with the Archytas' Curve determined by the intersection of the torus and the cylinder. Figure 4 b we see the point P, which is one of those four points, corresponding to the represented quadrant.

**Figure 5.** Point P becomes the starting point for the construction of a series of triangles, similar to each other. They allow to find the two proportional means we were looking for.

## Steps of the Construction. Determination of the proportional means

- Let APC' be a position of the generating circumference of the torus, with APC' a triangle inscribed in it, therefore, it is rectangle in P.

- Let M be the point where the cylinder's generating line that passes by P cuts the circle ABC.

- Draw the section of the cone with a plane perpendicular to its axis and containing the point B. BQE is the circumference obtained. The point Q is the intersection of the segment AP with the circumference.

- Points M and N are the perpendicular projection of the points P and Q on the plane ABC.

Then:
QN·QN = BN·NE = AN·NM (Euclid III.35 [5])

Where it results that AQM is a right angle. But APC' is also a right angle, hence MQ is parallel to C'P.

From Figure 5 let's extract the right angled triangle APC'.

**Figure 6.**

Since all right angled triangles in Figure 6 are similar, because their angles are respectively equal, it follows that:

$$\frac{AQ}{AM}=\frac{AM}{AP}=\frac{AP}{AC'}$$

But AQ = AB and AC'= AC where it follows:

$$\frac{AB}{AM}=\frac{AM}{AP}=\frac{AP}{AC}$$

This means that AB, AM, AP, AC are in continuous proportion so that AM and AP are the two proportional means we were looking for. In particular, the measure of AM coincides with the edge of the cube whose volume is twice the volume of the cube of edge AB. [6]

This shows that the solution exists if we are not imposed with the restriction to find AM constructing only with a ruler and a compass. [7]

## Short Conclusions

Throughout history, the solution has been sought under the conditions imposed by the Geometry of Classical Greece. Therefore, the solution given by Archytas does not satisfy these conditions. However, it is undeniable its merit implying an amazing creative capacity to produce a mathematically correct solution at that time.
So are the geniuses... beyond their time! We, the common people, should content ourselves just understanding their reasoning incorporating it as new knowledge to our collection.

## Notes

[1] Constructions with a ruler and a compass are made through a finite number of steps. They are based on the use of an ideal compass and an unmarked ruler. They were the basis to construct the forms in Greek Geometry.

[2] An example of mechanical solution is the division of a segment into equal parts with a marked ruler. However if it is done with unmarked ruler and compass it is perfect, geometrically speaking.
[3] Remember that in Mathematics a proof is a process by which, through a series of logical reasonings, you come to establish the truth of a proposition or theorem from a certain hypothesis. But a verification is to see that a certain hypothesis is true for cases where it is verified and one can not be sure if it remains true for any case.
[4] See Appendix 1.Proportional means between two numbers.
[5] In Appendix 2 we transcribe Proposition III.35 of Euclid, extracted from:
http://www.claymath.org/library/historical/euclid/ , visited on 04/16/1910.
http://rarebookroom.org/ visited on 12/04/2010.
[6] See Appendix 3.
[7] See Appendix 4.

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# APPENDICES

## Appendix 1: Proportional means between two numbers.

Given two numbers a and b, it is possible to find a third number x, called the proportional mean between a and b if it is verified that:

a: x:: x: b

(read as "**a** is to **x** as **x** is to **b**")
This is a continuous proportion, expressed in algebraic language as follows:

$$a / x = x / b$$

If between two numbers a and b can be two others x and y such that:

a: x:: x: y:: y: b

wich is expressed algebraically as:

$$a / x = x / y = y / b$$

Then x and y are two proportional means between a and b.

Example: **1: 2:: 2: 4:: 4: 8** written algebraically: 1 / 2 = 2 / 4 = 4 / 8. Then 2 and 4 are "proportional means" between 1 and 8.

## Appendix 2: Proposition of Euclid used in the development of the work

*Proposition III.35:* If in a circle two straight lines cut one another, the rectangle contained by the segments of the one is equal to the rectangle contained by the segments of the other.

## Appendix 3: On the similarity of right angled triangles

Two right angled triangles are similar when their two acute angles are equal.
Remember that the proportionality between the sides is determined from being opposed by equal angles.

*In this case QM//PC' and PM//QN, as it was shown.*

*This means that all right angled triangles seen in the figure are similar and so their sides are proportional.*

## Appendix 4: Demonstration of the impossibility of doubling the cube with ruler and compass.+

In the nineteenth century, Gauss concluded that in reality this is an unsolvable problem, but he did not prove it. As we already mentioned, the lack of solution comes from the demand to solve this problem using only unmarked ruler and compass.

It was Pierre Wantzel, from France, who in 1837 finally made public the corresponding theorem, in an article entitled *"Recherches sur les Moyens de Reconnaître si un Problème de Géométrie Peut se Résoudre avec la Règle et le Compas" ("Researches on how to recognize whether a problem in Geometry can be solved with the ruler and the compass").* It was published in the Journal de Mathématiques Pures et Appliquées.

He shows that the impossibility of doubling the cube with unmarked ruler and compass derives from another impossibility: to build with these instruments of classical Greek geometry the cubic root of any rational number.

http://www.uam.es/personal_pdi/ciencias/barcelo/historia/Gauss.pdf . Pp. 14 and 15. Visited on 24/04/2010.

CHAPTER 4

## Simulacra of *Nonorientable* Surfaces—Experienced through *Timing [1]*

William S. Huff

### Properties of the *Nonorientable* Surface

A "Möbius strip," identified in the mid-19th century by the mathematician whose name is attached to it, is frequently depicted in dictionaries. As the tradition about it goes (compared to a strict topological definition), a Möbius strip or band has one side to its surface and it has one edge—unlike the surface of a cylindrical strip, which has two sides (an *outer* side and an *inner* side) and two edges. Attention has to be given to the differentiation between two words: "surface" and "side."

The marvelous property of a Möbius's surface is called *nonorientability*. It has been pointed out that "the sides of [a closed surface as a cylindrical strip] could be painted with different colors to distinguish them" and that, since "the surface [of the cylindrical strip] has boundary curves, the two colors meet only along these curves" so that "anyone who contracts to paint one side of a Möbius strip could do it just as well by dipping the whole strip into a bucket of paint." (Newman 1956: 595)

Fact, however, intrudes: A Möbius, in its absolute, is two-dimensional. It is an abstract concept to be imagined. Unlike the face of a cube, it can neither be viewed nor touched —nor can it, in reality, be painted. When a Möbius is modeled with a strip of paper, substance is involved. That piece of paper has a measurable thickness; therefore, the concrete object takes on three-dimensionality. Such a material object, meant to represent a Möbius, actually has two surfaces—the specific surface of interest, the width and length of the strip of paper, and a consequent thin but measurable surface, the thickness of the paper itself. Furthermore the materialized object has two edges, which border the two surfaces. Though the materialized Möbius is a deception of sorts —that is, it is a simulacrum—it remains an intriguing object, which has been varyingly explored and aesthetically exploited by such an artist-sculptor as Max Bill.

If the paper Möbius's thickness is expanded to correspond to the width of the strip, it becomes a twisted, faceted torus (with a square cross-section). Since, as a thin piece of paper, the object at hand had been given a 180° twist, the object, as the four-faceted fattened torus, retains the 180° twist—and the object can, consequently, be painted in two colors. In order to occasion a square cross-sectioned torus, allowing only one surface to paint, the twist needs to be 90° or 270°—or $n$90°, where $n$ is odd.

## An Impertinent Juxtaposition

It is to be recognized that the Möbius—even though, as a rigorous topological artifact, it has zero thickness—can be deformed to any length (and relatively to any width, as well) and that, if it is greatly lengthened, it can be imagined to tangle into snarls. Furthermore, its single edge can be imagined to be smooth or jagged or a combination of both. However, if a *concrete* Möbius strip is fashioned with a suitably trim and firm piece of paper and kept within certain proportions—say, a width of one unit to a length of seven—it will physically take a *compact* shape that is textbook familiar.

Associating isometric symmetry with a topological artifact might seem to be a disciplinary contrariety, but wondering what kind of symmetry—if any at all—a Möbius strip might assume should not be ruled out of the bounds of a designer's natural curiosity. I put this question to one of my basic design classes, and a number of students quickly came up with the answer: The Möbius inherently possesses the potential of *twofold rotational symmetry*.

Due to the frequent orientation of a conventional depiction of the "Möbius strip," as it appears in *Webster's Collegiate Dictionary* (9th ed.), the twofold rotational axis is not visually obvious; it can, nonetheless, be picked up by an informed second look. In the depiction in Cundy and Rollett's, *Mathematical Models*, the twofold axis is more clearly evident.

(Figure 1)

These two depictions aside, the twofold rotational property should be conceptually obvious in consideration of the 180° twist that is given to a longish strip of paper before it is joined into the mode of the Möbius. With that question, so readily answered, my students were next set to the task of exploring the plastic potential of Möbius-like figures, as well as faceted twisted tori. Despite the absolute two-dimensionality of the strictly authentic, topological Möbius surface (i.e., zero thickness), its essential occupation of three-dimensional space was recognized as an imperative condition of the students' studies of nonorientable surfaces. Some extraordinary 3-D objects have resulted.

A clarification is in order: My design students were not urged to pursue new geometric principles, but to ponder existing ones. For example, in pondering the twisted, square-sectioned torus, it could be comprehended that, while such a torus—that is, a toroidal tube with zero-dimensional thickness—can have one continuous surface on the *outside*, if given the appropriate twist, it would, correspondingly, have one continuous surface *inside*. Such objects must, then, not be a genuine nonorientable surfaces: As surfaces of three-dimensional objects, they do *not* have the essential nonorientable property that the surface of the *Klein bottle* has. It might, then, be said that many objects that were designed in my studio skirted the fringes of nonorientable surfaces. This investigative excursion, did, however, throw light on the real geometry of the material Möbius strip, as covered above.

(Figure 2)

## The Experience of *Timing*

On previous occasions, I gave oral and written accounts of a type of design, regularly assigned in my basic design studio—*the parquet deformation*—which disposes *time* to participate as an integral *third* dimension, thus dynamizing the two-dimensional *spatial* content of the design. Commentary on the aesthetic potential of the parquet deformation was presented at the Katachi 2 conference (Huff 1994: 219-222), and commentary on its geometric requisites was presented at the SEMA 4 conference (Huff 2003: 9). I liken the *parquet deformation* to a remarkable art form, the Chinese handscroll, which, in its most exceptional, but younger genre, the landscape handscroll, goes back a thousand years. Time unfolds as the scroll is synchronously unrolled and rolled—pleasurable frame by pleasurable frame—not dissimilarly to how music flows. Time is engaged, however, in a different manner in respect to compositions whose three dimensions are all spatial.

During my early days of teaching basic design, I was well connected with many faculty members in Josef Albers's Department of Fine Arts at Yale, among whom was sculptor Erwin Hauer. A central precept of Hauer's teaching dwelt on the experiential potence of sequential events—which he called *timing*. Timing is transacted by the viewer's walking around a plastic object or by the viewer's otherwise successive change of position in respect to the object. Architecture cannot be fully experienced unless the viewer walks, not only around, but through it.

Working with a professional photographer, I recorded my students' best results from the several assignments that required three-dimensional modeling. Though timing was at the top of my students' stipulated considerations as they worked on studies of nonorientable surfaces and the other 3-D topics—originally, perhaps, neither the students, the instigators of the designs' essential formulations, nor I, the critic of the designs' aesthetic developments, were totally appreciative of the whole gamut of timing relationships that chanced in the best of these objects: When, however, the objects were once again given the scrutiny of the keen eye—meticulously freezing and capturing remarkable camera positions—the complete range of the metamorphic magic became evident.

A collection of *timing* sequences of eight studies of *nonorientable surfaces*, selectively registered by the camera at exceptionally diverse angles of vision, are presented here.

(Figure 3 to 10)

## Notes

[1] The text of this chapter is a revised edition of the paper presented at the *V National Congress of Sema*, in Resistencia, Chaco in the year 2005. Some concepts of the original translation of Marcelo Coccato were sustained while others were changed in this extended version.

[2]See an article that appeared eight years after my basic design students' first engagement with concretized Möbius bands and twisted prismatic tori: Martin Gardner, "Mathematical Games: A Möbius band has a finite thickness, and so it is actually a twisted prism," *Scientific American* 239 (August 1978) 18-24.

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**MARGINAL NOTE**

## Jewelry based in Möbius stripes


Darío Bessega


The Möbius strip as spatial surface does not have an internal or external face, nor front or back, because these properties are reversed along the stripe.

It is evoked in psychology, philosophy, art and design fields as a concrete starting point to understand the concept of infinity, because its representation alludes to continuity in a circular course, where the departure point is also the end.

My intention to use it to design objects has to do with an alternative view of geometry. In rings and pendants the strip solves functional requirements of each object, through its particular features: encircling the finger in the ring and providing a hanging surface in the pendant. The strip does not appear as an icon or a decoration applied on the object but the object is designed like a strip, alluding to the detour and not emphasizing a central place.

While the ring is used, the section of the strip that corresponds to the body of the ring is perceived as an “internal” face because it remains hidden. The half turn twist which identifies the Möbius stripe, goes from the end of the body towards the front, and appears as the interior of each side integrating itself to the other side, which is perceived as the “outside” or visible face of the piece.

In general, the use of the pendant suggests the presence of a rear side that rests against the chest. In the pendant design I suggest the rupture of this situation by generating an object without rear side, or with interchangeable front and back sides.

So, the object has more possibilities in its way of use, because it allows that its two “faces” that can be chosen as the visible face. Seeing the object from any of its fronts, the back side appears. Front and back dialogue with each other through the continuity of the strip.

CHAPTER 5

## Polyhedra: the source for twisted loop sculptures

Robert A.Wiggs

Some scholars who study polyhedra focus primarily on polygon faces and their structural relationship to each other. As we know, all polyhedra are named for the number of polygon faces in their anatomy. For example, the regular tetrathedron has four trigonal faces, the regular dodecahedron has twelve pentagonal faces.

Models of 3-D polyhedra are generally constructed of opaque material. This presents a viewing problem because no more than one half of a 3-D polyhedron may be seen on any viewing axis. There are many interactions that cannot be seen in the opaque models. The regular octahedron below demonstrates those interactions. When viewed as transparent wire model, the hemispheres of the polyhedron can twist and untwist in relation to each other. Some can be seen twisting on one viewing axis and some twisting on two axes. None of this interaction can be observed in opaque form.

(Figure 1)

In figure 1A the opaque model octahedron is composed of faces, vertices and edges. In its 4-fold mode, there is no twisting that can be seen on the viewing axis. In figure 1B the transparent model of the same octahedron has the structural components of an equatorial wavy ring and trigonal caps that are twisted in relation to each other on its 3-fold viewing axis. Figures 2A and 2B are exo and endo models of the transparent octahedron without faces, vertices, and edge lengths. Figures 3A and 3B are curvilinear models of the transparent octahedron without faces, vertices and edge lengths. The coordinates for 2A, 2B, 3A, and 3B are the same as the regular octahedron, however, they circumnavigate continuously around their spatial coordinates.

(Figure 2)

This sculpture, carved in wood, has a 3-fold twist viewing axis. It is composed of twists and hyperbolic saddle surfaces between the twists.

Research by the author has led to the discovery of the ninth self all-space filling prismatic polyhedron he named "Twist Octahedron". This research generates also sixteen families of polyhedral lattices and is the source for many pieces of twisted loop sculpture.

**MARGINAL NOTE**

Patricia Muñoz

Claudio Guerri and William Huff introduced me to Robert Wiggs and his son and permanent collaborator, Calvin. So, I got to know these marvelous spatial lines, which no one can doubt they are 3D configurations, even if they are composed by arcs of circumference.

It was a challenge to build them. A fruitful and intense exchange of emails, carrying words, sketches and papers allowed us to produce these images that we can share today. I was fascinated by Robert Wiggs´ understanding of space, of the lines and the surfaces he uses in his sculptures, lacking the digital instruments of visualization and analysis that has made our task considerably easier.

## Exostructure of the Octahedron

Figure 1. Exostructure in the octahedron, standing in a triangular face, twisted loop with and without the polyhedron. The section that is parallel to the base, at half height, defines a hexagon in the octahedron that divides the spatial lines in its planar parts.

Figure 2. Twisted loops of the exostructure of the octahedron. A vision in the octahedron and the exo-structure on its own. The lateral projection evidently shows its continuity.

Figure 3. Polyhedron materialized through its edges and spatial line built using arcs of circumference inscribed in the triangular faces.

## Endostructure of the Octahedron

Figure 4. Endostructure: Structure with line and with the polyhedron.

In order to define the structure, the edges of the triangular horizontal faces (the one resting on the floor and the one on top) are connected inside the polyhedron by straight lines that touch a hexagon contained in the midplane. The twisted loop is composed by arcs of circumference inscribed in the triangles of the structure, connecting the superior and the inferior sectors with arcs that lie on planes defined by the structure.

Figure 5. Arcs of circumference that compose the spatial line

Figure 6. Endostructure: spatial line with structure and with the polyhedron

Figure 7. Line of endostructure and twisted loop

Figure 8. Simple and double twisted loops on the exo and endostructures of the tetrahedron.

For more information on this topic and an extended explanation we suggest the visit of the site http://wiggspolysutures.com/

## **PART 2.** The research project

CHAPTER 6

### **Spatial lines in projecting activities: Graphic Design**

Nora Pereyra

It seems to make no sense at all to talk about Spatial Lines in graphic design, but it is a recurrent subject in current designs not only in the branding but in editorial, web or multimedia design as well.

The issue is that graphic designers think more on images, movement, interrelations, chromaticity, textures than in the generative process that allows the concretion of a line, considering both plane and spatial instances, or its controlled transformation of attributes involving families and series of figures. These topics, together with symmetry parameters and the above mentioned inter-relationships are used to materialize current designs. They can be transformed, leaving aside the static characteristics of other periods.

Digital media and the wide range of variations that software enables, make the presence of spatial lines possible in almost all graphic interventions, no matter if the designer knows about its generation and controlled transformation. In most cases what the designer is looking for, I dare to say, depends whatever software and imagination can provide, together with aesthetic parameters. It is conditioned too by the perceptual proposal. It could be said that years of acquired expertise through professional training and practice guide the decisions that finally define a design.

It is obvious to me, but in a world where the dominance of image is support for a great deal of social relations, the complexity of symbolic implications demands an image development according to these conditions. For instance density, suggested solidness, dynamism, network communication or interrelations are based on corresponding images that permanently grow or mutate. These modifications are usually understood, among others, as adaptations, revisions, rapid comprehensions that find their graphic expression, where spatial lines take an important place. Regarding their manipulation, the chance to work with them and their transformations optimizes system variables.

Perhaps this presentation can be judged as arbitrary or capricious; yet for a long time and according with current requirements, symbolic value is the one that emphasizes and surpasses the communicational level of a graphic product that exceeds what is merely referential or at least, it does so once it has reached the visual consumer. I have used this term, not in a naïve way because at present, we all are voracious consumers of images and visual information, whether we approach them to deconstruct and elaborate them, to discard or consume them as merchandise with use value.

Up to this point, we can begin to see some uses of spatial lines on graphic applications in different kind of products. Materialization can be considered representational as far as it reproduces something that exists or that suggests a future existence in objects. Also, as elements of direct materialization from its drawing.

## Spatial Lines in Design products/presentations

In most cases, the way of dealing with spatial lines derives from overlapping conic curves, but mainly they are obtained from the sphere: spherical or conical helixes, cichloids and spirals projections and/or their transformations. So, we have simple applications, as the one shown in figure 1: information graphics; in figure 2: Vangelis compact disc cover —¨spiral¨ is its name –; or figure3: one of many alternatives designed by Garrido-Reissis studio for a logo.

The use of digital technologies expands our vision and allows –as in any design as well, the establishment of new visual paradigms: more sophisticated, with high aesthetic level and various alternatives of interpretation. Such as a Neville Brody poster that can be seen in figure 4 and the front and back cover of Why Not Associates Studio book in figure 6. We can find major complexity and linear intersections in Neville Brody's type in figure 5, or in Estudio Cabina website, figure 7, which create spatial surfaces.

Of course, these are planar representations of spatial situations with different degrees of subtlety. Strong directionalities can be seen; overlays and transparencies in addition of a wide range of chromatic and contrast variables.

Figure 1 Information graphic. http://www.formlessmountain.com/agal.htm

Figure 2: Vangelis CD cover

Figure 3: Garrido Reissis Studio.

Figure 4: Neville Brody. Fuse 98. Poster

Figure 5: Neville Brody. Made in Clerkenwell. In the site: researchstudio.com

Figure 6: Why not Associates. Front and back cover of their book Why Not. 1998

Figure 7: Estudio Cabina. Website. Background patterns. www.espaciocabina.com.ar

Frequently, materializations are not restricted to lines and rarely describe their morphogenesis, but refer to objects, as the example on Vangelis CD cover. But, as the other illustrations show, lines concrete virtual situations, abstract drawings that provide spatial effects to the graphic space treatment while enhancing the value of the graphic sign itself. This kind of concretions is usually associated to high technology, up–to-date, modernity, speed or efficiency conditions. They are omnipresent in complex graphic systems solutions.

It is relevant to note that, in general, spatial lines coexist with flat graphic areas, where text blocks appear, integrated and understood as natural. Imagination and new visual and interactive paradigms encourage the simultaneous appearance of many representational modes that are read without contradiction. Such is the situation of real-time movement on websites, diverse options at every click or the volume of architectural and environmental graphic applications, in addition to those already explained.

These statements do not close the treatment of spatial lines in graphic design. It has just been one possible approach in the world of graphic design shapes.

CHAPTER 7

## Spatial lines in projecting activities: Industrial Design

Patricia Muñoz

Spatial lines have found different ways of finding their place in design. Their possibilities of implementation in products are linked to the development of the necessary technology for their materialization. While analyzing industrial design products we have noticed three different modes which we will proceed to describe. However we want to remark that while we were carrying out this research we were able to verify the little diffusion of industrial design objects made in Argentina. We hope this situation will change soon because we have much to learn from our professional local production.

The different approaches are:

- Materialization of the line
- Spatial lines as borders of bigger surfaces
- Spatial lines as edges

### Materialization of the line

The more numerous strategies are linked to tube bending, working with its total or partial curving. The production of planar parts that are welded to construct spatial lines contributes to its extended distribution. Classic examples of this use are the Wassily chair (1925) designed by Breuer, the Cantilever chair (1928) designed by Stam and the Cesca chair designed by Breuer (1928). Among local production it is remarkable the structure of the BKF chair (1938) designed by Bonet-Kurchan-Ferrari Hardoy and the W chair, designed by César Janello (1946). In this furniture, spatial lines are conformed by straight tubes connected with curved sectors that provide continuity.

Figure 1. Spatial lines materialized as structures of seating furniture: Wassily chair (Breuer 1925), Cesca chair (Breuer 1928), BKF Chair (Bonet, Kurchan, Ferrari Hardoy 1938) and W Chair (Janello 1946)

There is another mode of materializing them, such as the Littlebig Chair (2006) designed by Jeff Miller, which highlights the line, while disturbingly separating it from the body of the chair. This strategy makes more evident its three dimensional character.

Figure 2. Spatial line as structure and handle of the Littlebig Chair, designed by Jeff Miller for Cerrutti Baleri. Photograph of Ezio Manciucca

If we think on the line in a more perceptual than geometrical way, we can consider the Leaf Lamp (2006), designed by Yves Béhar, as the concretion of a spatial line. One of the dimensions prevails over the other two, and the detail of the central discontinuity emphasizes this interpretation.

Figure 3. Spatial line materialized by curved areas in the Leaf Lamp, designed by Yves Béhar for Herman Miller

Another way of using these shapes in products is through the connection of planar fragments, such as the system A3, designed by Asymptote, in which the edges of the screens —planar lines —define spatial lines in their combination. There is material discontinuity among the parts, but the morphological treatment produces a clear comprehension of the line continuity, as it can be seen in the upper edge of the workstation shown in Figure 4.

Figure 4. Spatial line produced by the combination of planar parts in A3 System, designed by Asymptote.

Spatial lines can also appear as units that create a bigger surface through a systematic constitution, such as the helices that materialize the cylinder of the base of the Spin Table, designed by Escalona.

Figure 5. Spatial lines as units of a systematic constitution in the base of the Spin Table, designed by Joel Escalona

At this point it is necessary to include further explanations. According to the Doberti´s System of Figures, the shapes that belong to this typology should be continuous. However, some concretions can be understood as a continuous spatial line even if they present geometric discontinuity of curvature. For instance, the Zig-zig chair (2007) of Michael Malmborg clearly shows this tension between the continuous path and the interruptions provided by the changes in directions. The square section makes these twists more evident. In addition, in two of the turns, there is a separation of the soft and hard zones of the object.

Figure 6. Discontinuous spatial lines, of square section, in the Zig-zig chair, designed by Michael Malmborg for Lyx

## Spatial lines as borders of bigger surfaces

Spatial lines can also appear in products as boundaries of sectors of bigger surfaces, for instance the headlights in cars, which are strongly defined by the shape of their borders. The morphological treatment of the body of the car influences this definition.

Figure 7. Vehicles with different limiting line of the headlights through spatial lines

In addition, these shapes can indicate discontinuities in the same surface, to organize the disposition of controls in an instrument panel, or to define functional areas. Different uses can be combined in the same product, as in the lid of the container shown in Figure 8, whose edge and whose opening area are defined by two different spatial lines.

Figure 8. Two spatial lines, one as edge of the lid and another indicating the opening area.

Another clear example is the Aspen Friendly nebulizer (2002) created by Punta Diseño, that shows a limiting spatial line between the transparent lid and the case that plays an important role in the design project.

Figure 9. Spatial line as border of the transparent lid of the case of the Aspen Friendly nebulizer, created by Punta Diseño

An interesting example of the use of these lines as discontinuity between surfaces is the Miss washbasin, of Meneghello Paolelli Associati, which breaks the traditional conception of sink and pedestal, integrating them in a different way, keeping the opposition front-back. The shape is geometrically and materially continuous even if perceptually two different spatial surfaces are identified, limited and simultaneously connected by the spatial line.

Figure 10. Spatial lines as limit between surfaces in the Miss washbasin, designed by Meneghello Paolelli Associati

## Spatial lines as edges

Spatial surfaces are limited by plane or spatial lines. There are diverse products that highlight this feature. Other objects are full or partial materializations of helicoids and Möbius bands, whose edges are clearly defined.

In this group, among the national production, we can include the Placentero Chair (2006), designed by Diego Battista[2]. The spatial line lies on a sphere and determines the discontinuity between the rigid shell and the padded seat.

Figure 11. Placentero Chair, designed by Diego Battista

Spatial lines can be recognized even if they are made of small segments, such as Marco Dessí´s radiators [3]. We clearly perceive these lines by the continuity of direction of the thickness of the rotated square components.

## Spatial lines and digital fabrication

Digital fabrication systems enable the materialization of very complex shapes. We can find spatial lines, conforming objects with variable thickness, contacts and divisions as in the Edge pen [4], designed by Lovegrove. This can be appreciated too in different lamps manufactured by 3D printing, such as the lines of Bathsheba Grossman's design Flame, MGX.

Figure 12. Flame.MGX lamp produced by MGX by Materialise, designed by Bathsheba Grossman

## Some conclusions

Throughout these observations, the presence of these shapes in Industrial Design objects is clearly noted. In different manners, they are part of our everyday context. They are complex forms, which speak of three dimensions using a minimum amount of material and they make evident the treasures and the paradoxes of their features.

Ezio Manzini (1993) states that design is in the meeting point of what is thinkable and what is possible. The progress in the capabilities to control, model and visualize these shapes promoted new ways of understanding them. The technological advances allow new materialization alternatives. However, the acknowledgement of morphogenerative strategies is equally important. It enables innovative explorations in applications beyond what is already known, making the project with spatial lines possible, surpassing the forcefulness of their concretion.

## Notes

[1] It is part of the Argentine Industrial Design Collection of the Museum of Modern Art of the City of Buenos Aires

[2] First price in the "Industrial Design" category of the contest Innovar 2007, organized by the Ministry

of Science, Technology and Productive Innovation

[3] The radiators can be seen in www.marcodessi.com/ visited 30/04/2010

[4] http://www.unicahome.com/products/small/35548.8F66FF31.jpg visited 30/04/2010

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CHAPTER 8

# The research project

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## Introduction

This work is a derivation of what was explained in Chapter 1. These initial steps became a formal Research Project [1] whose aim was to comprehend and to analyze the characteristics of spatial lines. This typology constitutes an attractive and interesting area of morphology which had not been thoroughly examined. It is a relevant field for design, in particular Industrial Design, as we have already shown in Chapter 7. The laboriousness of their representation and the difficulties to materialize them, promoted a limited conceptual development. However, at present we have adequate digital media for their visualization, analysis and fabrication; and an extensive morphological knowledge on spatial surfaces, which make their rigorous study feasible. We consider that the generation of forms for products should not be whimsical or fortuitous because we understand them as a place of synthesis of functional, technological and communicational factors. The morphological definition of a product requires much more than a moment of inspiration. It demands the knowledge that enables its intentional handling. This is the reason why form control in this kind of figures is particularly relevant for their application in product design.

### Types of spatial lines

In this exploration three families of spatial lines were clearly identified.

1. Spatial lines created by the intersection of spatial surfaces
2. Spatial lines created by selection and combination of planar sections of spatial surfaces.

3. Spatial lines as continuous combination of planar curves inscribed in faces of polyhedra.

Figure 1. The three types of spatial lines detected

### Lines as intersections of spatial surfaces

These are those obtained as intersections of spatial surfaces. In certain dispositions, planar curves can be obtained. For instance, in Figure 2, the intersection of a sphere and a torus is shown, in which the sphere is tangent to the torus in its horizontal maximum and minimum circumferences. This intersection produces two crossed Villarceau´s circles [2]. Moving one of the figures away from the other, the intersection determines spatial lines.

Figure 2. Detail of the spatial relations between the torus and the sphere, and the resulting lines

### Lines obtained by selection and combination of planar sections of spatial surfaces

We can also design spatial lines composed of planar sections of spatial surfaces. For instance, in Figure 3, a spatial line is built combining selected fragments of the intersection between a triangular prism and a sphere. The spatial line is formed choosing sectors of the three circumferences obtained. These type of lines are less continuous in their curvature than those created by direct intersection of spatial surfaces. These lines present tangent continuity (G1) but not curvature continuity (G2). However, the possibility of composing spatial lines from planar fragments has a positive side, as it makes easier its spatial manipulation, as it is possible to work with the planes that inscribe each sector.

Figure 3. Detail of the spatial arrangement of the planes and the sphere that enables the construction of the spatial line.

### Lines as continuous combination of planar curves inscribed in faces of polyhedra

It is possible to work with the combination of arcs of circumference and ellipses tangent to the edges of the cube, as it is shown in Figure 4. These lines have tangent continuity (G1) on the edge in which the joint lies.

Figure 4. Spatial line built as planar curves joined in the cubic structure.

### Transformations

A series of transformations were produced, based on the attributes of these lines, forcing the limits of their identification.  In the first group –spatial lines as intersection of spatial surfaces- the original operands were modified in their proportions and in the spatial disposition related to the original shape.

The exploration which modified distinctive aspects of the operands provided a new comprehension of traditional geometric shapes, which allowed us to understand them as particular instances of continuous transformations.

For example, in Figure 5, the isometry of a bi-cylindrical curve can be transformed moving one of the cylinders or increasing its diameter, or changing its proportion to a conic shape. In this last instance, the lesser gradient of the generative line corresponds to a greater transformation of the spatial line.

Figure 5. Transformations of the intersection curve of two tangent cylinders with different diameter.

The modification of the spatial relations between the figures produced continuous sequences of curves, regulating the overlapping area. The translation of the axis of one of the operands resulted in new series of spatial lines. This is illustrated in the series between the sphere and the elliptic paraboloid shown in Figure 6. The first case produces two secant circumferences.

Figure 6. Transformations of the intersection line between an elliptic paraboloid and a sphere.

We were able to check the direct influence of the spatial relations and of the transformations of the operands in the determination of spatial lines. This made the control of changes easier, and provided the possibility of operating intentionally with these shapes.

Another relevant factor in the relation of spatial lines and the figures that determine them is that we can define the former considering the properties of the latter. For instance, the line shown in Figure 7 is the result of the intersection of a cylinder and a sphere, so it is a cylindrical *and* a spherical curve.

We could define this spatial line as the set of points that satisfy three conditions:

1. They must have a constant distance (R) to a point (centre of the sphere)
2. They must have an invariant distance (S) to a line (axis of the cylinder)
3. The distance from the point to the line should be greater than 0 and smaller than R+S

Figure 7. Definition of the intersection line between a sphere and a cylinder considering the properties of both shapes

The transformations we have previously described were also applied in the second group, but new alternatives were considered. When points of double tangency appeared, more than one line could be obtained, according to the partial selections made. This generative strategy was relevant for the design of spatial surfaces.

In the third group, transformations were fundamentally defined by the polyhedral structure. When it was altered, it changed the construction frame for the line. The line could also modify its symmetry by moving its relevant control points within the structure.

## Generation of spatial surfaces

When we created spatial surfaces with spatial lines we defined three different strategies. They were used in the courses of Morphology and are explained in detail in the next chapters.

Figure 8. Spatial lines as generative lines (a) or as paths or directing lines (b)

Its main features are these:

### Spatial lines as generative lines

Spatial lines define the surface through a regulated movement, with the possibility of changing throughout the path. Two factors should be considered in order to avoid crossings: the distance between generative lines and their scaling.

**Spatial lines as paths or directing lines**

Spatial lines determine the path for the generative lines. The main variations deal with the shape of the generative line. At first, straight lines were used, which were later replaced by curves or became axis of other generative lines.

**Spatial lines as edges of compound surfaces**

The use of these lines to create new shapes starting from known forms enabled the creation of perceptually complex configurations which were simple in their component parts. The use of surfaces of union (fillets and blends in CAD systems) provided a tool to control the integration of the different segments.

## Some conclusions

This work has established a two way relation. Outward, in the development of interesting spatial lines that could create innovative spatial surfaces. Inward, in the progress of the knowledge of well know shapes as the sphere, as we have discovered new creative possibilities which were only made evident by the interaction with other shapes, in the imprints and marks left on the original shape. We could also build transformation sequences, which allowed us to make intentional alterations in the search of the shapes we imagined [3].

The knowledge produced in this research on spatial lines has given us control elements which enable intentional handling of these shapes, which is necessary to introduce them in industrial design objects. Its aesthetical value is increased with their possibilities of regulated variation and their morphogenerative potential.

Digital instruments have played an important role in this inquiry. We agree with David Perkins [1997:150] when he states: "It is very frequent that technologies present themselves as a cognitive sandbox, and that they propose to build whatever is possible within it." We understand that this excess of exploration weakens its potential. One of the more relevant aspects of the use of digital media in design is its instrumental character for the research of complex shapes, in their analysis and in their adjustment for manufacturing. These technologically mediated actions surpass mere operations and become a research method for the analysis of forms.

The previous knowledge on this typology, increased by the research produced with strong digital support, allowed us to introduce this subject matter in our courses of Industrial design undergraduate program at the Faculty of Architecture, Design and Urbanism, at the University of Buenos Aires. An experience carried out in the three courses of Morphology will be described in the following chapters.

These shapes are a challenge, not only because of their complexity but because they are composed forms: they are the common edge of different figures which have diverse attributes and, as any limit, they separate and link. These lines confront us with shapes that do not fit in unique classifications and question some concepts we already considered settled. In spite of this, it is worthwhile dealing with them because their knowledge opens new and attractive design possibilities.

## Notes

[1] This work was developed as a research project (code SI MYC07) Spatial Lines: determination and production. Director: Patricia Muñoz. IEH, Laboratory of Morphology, SI, FADU, University of Buenos Aires (2007-2009)

[2] Villarceau´s circles can be obtained as a planar section of a torus with a plane tangent to two opposite generative lines (in the rotational generation)

[3] Some results of these explorations are shown in the addendum

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# **PART 3.** Educational explorations

CHAPTER 9

## The implementation project

Patricia Muñoz

The institution in which we carry out our research is the Faculty of Architecture, Design and Urbanism, which includes five Design programs: Graphic Design, Industrial Design, Textile and Fashion Design and Image and Sound Design.

It is a public University, free of charge for students, and has massive population. In the experience we will describe, the teaching staff was composed of eighteen professors who worked with three hundred and fifty eight students, divided in several groups.

There were a hundred and fifty six students in the first course — Morphology — distributed in seven groups with a coordinator, a hundred and two students in the second —Special Morphology 1 —with four groups and a coordinator and a hundred in the third one —Special Morphology 2 —with four groups and a coordinator. In addition, there is a general coordination that I carry out with the collaboration of one of the Assistant Professors.

Each annual course has thirty meetings of four hours each, once a week. The experience we will describe was carried out in a semester, so there were fifteen meetings. The three courses work simultaneously in three contiguous classrooms. This allowed an easy communication among the groups of different levels. There were no computers in the classrooms; we only have digital projectors for the lectures. This scarcity made us plan their use with anticipation and so it was not possible to use projections to explain something that emerged in the practice of the workshop.

This pedagogical experience is connected to the research project we developed, since 2006, in the field of Morphology for Industrial Design. The Project, SI MyC07, which was explained in the previous chapter, was developed in the IEH, Laboratory of Morphology, depending of the Research Secretariat, of the Faculty of Architecture, Design and Urban Studies, University of Buenos Aires. We consider it is necessary to make public the findings of the research projects, so that they leave the laboratory and go to the classroom. From the more protected place, of reflection and conceptualization, to the multiple and varied universe of the students that, in this assignment, verified and increased the initial knowledge.

The topic of spatial lines was undertaken from the specific objectives of each course, establishing connections to professional practice by means of the analysis of products that showed their relevance and appropriateness. Three different perspectives, corresponding to each course, allowed us to develop different aspects of the topic, which were integrated in some joint practices. We could understand the common topic as a narrative line, where contents were bound in a common construction, throughout a shared period of time.

### Instructional sequence: separated and mixed

The joint work of the three courses on the same subject got through different practices connected by an instructional sequence. The courses began with a practical approach to the subject of spatial lines, in which each group produced objects which, in turn, became instructional material for the following assignments. A general lecture followed, where the topic was presented to the students of the three courses. Later, each course worked separately, even if they shared some lectures and visited the different courses, especially on days when partial advances were exhibited. By the end of the term, the three courses made the same practice, transferring to an industrial design product what they have learnt. In this last assignment the students of the three courses worked mixed in the same classroom. This was very interesting because they could share and see different points of view and comments were more varied. Finally a closing lecture took place, which recovered the initial explanations with examples of the students´ work. There was also a comparison between specific issues which emerged in the different courses

CHAPTER 10

## Exploring form and teaching of Spatial Lines in Morphology

Nora Pereyra

In the first level of Morphology at Industrial Design undergraduate program, spatial lines were studied according to students´ knowledge and in a general view. They have already gone through an introductory course and they have to face specific knowledge and the vocabulary that they will learn along the assignments.

The tasks of the students in this first course aim to add instrumental and conceptual resources to work intentionally with forms, discovering the different attributes that can be transformed in order to create alternatives and new projects. The final result of a practice was reused as starting point to develop new projects. In the first Morphology four-month period we worked with a classifying system of shapes and its ways of concretion; with continuous and discontinuous languages; transformations; drawing methods and presentation techniques. All these subjects were undertaken with Spatial Lines. The previously mentioned topics are especially important in Industrial Design because they are communicational resources with strong influence on perceptual factors.

On the first day of classes a first approach to the topic of Spatial Lines took place. We usually receive our students with a joint class. The models made up by each team were of the kind of spatial lines that they would be working on the four-month period to come.

Figure 1. Results of the team work in the first meeting

At first the typology of spatial lines was characterized. They were obtained combining planar lines placed on the cube faces. It should be noted that these are not the typical spatial lines described in *Sistema de Figuras (System of Figures)* (Doberti et al. 1971). So these spatial lines originated in the abstract structure of the cube, as a result of the continuous union of curves. We refer to physical and directional continuity, not to continuity of tangents and curvature. Not less important and always present, the concept of *interpretation* led all the practice development.

We will explain the relations emerging in every action taken while developing the work, which were mainly explained in the purposes of the students´ guide.

In the first stage, students concreted, using the saturation mode, a given spatial line based on one of cube interpretations, as shown in figure 2. Thus, students were introduced to disciplinary vocabulary and to concepts present in their work, such as *abstract structure, figure typology, and modes of concretion.* Every project decision was based on pairs of opposite ideas that defined it.

Figure 2. Spatial line, organized on the cubic structure, considering the cube as two trihedrons. Student: Rossi.

On a second stage, students modified the concretion mode with selective criteria, in order to propose different shape alternatives, either confirming or rejecting the original interpretation of the shape.

Figure 3. Volumetrical original concretion and alternatives of materialization. Students: Lucchese, Marino, Potente, Pratolongo

The next practice began with the analysis of one (of the) proposals previously produced in order to define equivalencies and differences in the attributes that determine the families of shapes, according to the System of Figures.

Figure 4. Analysis according to the System of Figures. Students: Lucchese, Marino, Potente, Pratolongo

Later on, intentional and selective transformations were made, based on decisions derived from the previous analysis. A significant change regarding the original figure was required, as can be seen in Figure 5. Students analyzed the transformed designs according to the System of Figures and compared it with the team's first analysis, in order to recognize what attributes were constant or variable among the group's projects. The ultimate aim was to become aware of the following purpose: to *verify the possibility of transforming systematically a project, generating alternatives through the operation of the shape attributes.*

Figure 5. Selective transformations. Students: Lucchese, Marino, Potente and Pratolongo; Sauri, Vacarezza and Colombo

At the joint exhibition of the workshop results, the aim was to make the students notice how the focus of attraction or the main opposition changed; or how a project confirms the original shape while other denies it; or how one of the opposite pairs was enhanced. In short, how transformation makes possible new ways to comprehend a form enabling recognition of its first identity. In the signification level, the aim of this action was to *recognize criteria underlying the transforming procedures*.

The next practice worked on the application of *discontinuity* to highlight different sectors of the designed shape. Components or areas were subtly defined, producing new interpretations of the starting shape, as it shown in Figure 6.

Figure 6. Two alternatives of discontinuity. Students: Lucchese, Marino, Potente and Pratolongo

Finally, 3D models were made. Different variables of understanding a shape, developed along the four-month period, are clearly shown.

Figure 7. 3D models of some of the projects of the Morphology Course, 2007.

Tentative projects were developed and adjusted by the students before they reached the final solution, guided by the professor in charge of each group. Throughout the practice described, multiple representations were used in order to strengthen the particular communicational possibilities of each instance.

**What was encountered throughout the practice that we had not foreseen?**

Along the development of this first term practices, there were relevant instances -on the process and in the final results- that should be pointed out because they expand the comprehension and operation on forms.

• Knowing the attributes of forms enabled our students to transform shapes in order to create alternative proposals, understanding that the use of ruled transformation gave them the chance to work on the relation confirmation-rupture as an innovative factor.

• As professors we widened our views in shape dealing, based on the diversity of proposals which emerged, accepting new transformation parameters after reaching agreements in the whole teaching staff. Beside it allowed a thoughtful understanding of instructional objectives and stretched operative and communicational bonds. As a result our teachers' commitment was deepened.

• Students understood that the same topic, developed in three different courses, with degrees of complexity corresponding to the learning possibilities of each level, offered rich and multiple alternatives that did not exhaust its knowledge, but opened a wide range of possibilities. This strategy was also understood as a method that could be used in other morphological matters, as design tool and as a way of thinking and operating forms.

## Spatial Lines in Special Morphology 1

Damián Mejías and Leonardo Moyano

The initial class of the workshop consisted in the construction of different geometrical shapes of great dimensions, materialized in cardboard, of scaled templates of the sections of the final object, as can be seen in Figure 1. So, after the assembling, students got to know how they could obtain some of the spatial lines as a continuous combination of planar lines.

Figure1. Spatial lines as combination of planar sections

This assignment was the starting point for the next practice where spatial lines were created. They were used to design spatial surfaces, either as generative line or as directive line. Later, transformations were made to create some volumes which were concreted in the end.

In order to restrain the complexity of the shapes used, students could only use planar lines on faces of surfaces or polyhedra, which were combined to create spatial lines. This exercise had different stages and was done in groups. There were instances in which each student designed a different form and others in which the whole group worked in only one alternative. So, learning was expanded because different projects were explored and, in addition, the adjustment and extensive development of a form was also produced.

The aim was to work with complex shapes, in particular spatial surfaces, dealing with the specific contents of the course, such us generative systems of surfaces with constant lines. If students used variable generative lines, they should also learn how to control this variation. Concepts of tangency, curvature and inflection were also included. The double role of planar sections of surfaces was recognized: as constitutive elements of given surfaces and as generative elements of new shapes.

In a first stage, work begun with known surfaces. Students created spatial lines combining planar sections. These spatial lines should be organized considering some opposite pairs of concepts such as continuous / discontinuous, homogeneous / progressive curvature, or spatial oppositions. At this stage, the line should have isometric symmetry. The joining points of the curve should be continuous but the spatial lines could include some angular points.

The use of planar lines was fundamental to control the complexity of the design process and to precise the spatial location of its points on the original surface, polyhedron or prism. Some of the planar lines were conics: circumference, ellipse, hyperbola and parabola. In the following example the student combined different sectors of ellipses and hyperbolas to obtain her spatial line. Different proportions were explored in order to choose the line which would be more suitable to continue with the project.

Figure 2. Variations in proportions of the conic surface which originated the spatial line. Student: Natalia Hoz

Figure 3. Spatial line combining two sectors of ellipses in the conoid, linked y a fragment of the conoidic

hyperbola. Student: Gabriel Mansilla.

In Figure 3, the spatial line combines sectors of ellipses inscribed in horizontal planes, joined by fragments of conoidic hyperbolas. The final shape has areas of different curvature, and interesting oppositions for the design of new spatial surfaces in the following stages of the practice.

One of the spatial lines created in the group of students was chosen for the next stage. It could be used either as generative line or as path. If it was used as a path, simple curves or straight lines were used as generative lines. In order to design the surface, the operations of rotation, translation, or radial displacement could be used. The generative lines could be homeometrically transformed.

In Figure 4 the spatial line, with transformations, is the generative line of the surface. A sector of an ellipse is the path for the movement of the line.

Figure 4. Generation of a spatial line. Student: Natalia Hoz

In Figure 5 there is another example of the first sketches, in which the spatial line is the directive line, or path, and a parabola with transformation is the generative line.

Figure 5. Sketches of different spatial surfaces starting from the same spatial line. Student: Rodriguez

Figure 6. Spatial line and surface created with the curve as directive line. Student: Ferriccio

Later, students modified a sector of the surface designed in the previous stage, creating a volume. The original generative or directive curves were transformed into a planar surface. This could be easily done because the spatial curve created in the beginning was composed of sectors of planar curves. So, students were able to control rigorously the modified area, and did not alter the original generative system. This transformation is catametric, because the introduction of a new typology is a breaking point in the continuity of transformations.

Finally, students designed the concretion of the form obtained in the previous step. The oppositions we have already mentioned, reappeared in these operations. The concepts used in the design of the original spatial line were used again in the materialization. The line was not just the beginning of the assignment but was built in different ways, showing its generative potential.

In the project shown in Figure 7, the original spatial line was recovered, integrating it in different ways to the rest of the object. In the alternatives A and B, the areas which generated the volume are fully materialized and are linked to the spatial lines, which in turn are defined as tubes. In the project C, the transformed areas are materialized as tubes and some sectors as volumes. This makes the object visually permeable, while in the other alternatives the opposition abstract/concrete is more relevant.

Figure 7. Surface with a volumetric area and three alternative concretions. Student: Hoz

In the other project, shown in Figure 8, the spatial line is materialized as a tube. It creates the surface by rotation with homeometric transformation.

The oppositions abstract/concrete and inside/outside are evident in the materialization of the areas transformed in volumes. The contact between tubes and volumes produces a balanced set and gives hints of the creative process.

Figure 8. Concretion of a spatial surface. Student: Sarra

The projects presented on Figure 9, show different alternative concretions of the same form, focusing in different attributes of the original shape.

Figure 9. Alternative projects for the concretion of a form. Student: Maschio

Finally, in the model of Figure 10, the original spatial line can be seen as fundamental part of the materialization of the object. The line was built with tubes of square section, visually rebuilding the surface through a systematic constitution. The upper and lower areas have greater compactness: curved surface and volume, in the endpoints of the spatial lines.

Figure 10. Model of the materialization project. Student: Lauría

## Some conclusions

In this practice, students could recognize the double role of the planar sections of the spatial surfaces: as generative system that builds and unveils the original surface and as possibility of creating new forms. In this line of thought Doberti (1989) says:

*"The marvelous "multiple significance¨ of the surfaces, which contain and hide at the same time all of its generative systems, that unfold differentiated, in front of the view which searches for comprehension whilst it synthesizes them in its unity and configurative continuity. Surfaces that are an ¨explanation¨ of the lines which furrow them and the ¨results¨ of the spatial organization of the multiple lineal systems that constitute them."*

Even if it sounds contradictory, it is right to say that it is important to know about planar curves, such as the conic lines and their possibilities of combination in 2 and 3D space in order to design spatial lines. Also, it is relevant to be able to recognize the categories that organize them and their attributes because it allows us to establish relations between them which confirm or question the forms they produce. Throughout the practice, families of spatial lines were detected, which made possible the definition of groups based in their proprieties.

Students worked, not only with shapes but with the structures that support them as well. The operations on the planes which inscribed the planar sectors of the spatial lines and on the geometric elements of surface generation and on the control of transformations, gave control elements of continuous shapes, enabling the transference of these kind of products to the universe of everyday objects in industrial design.

New morphogenerative strategies for spatial lines can be developed with these tools, keeping a rigorous control of what is designed, which is not opposed to a great creative production.

## Spatial lines in Special Morphology 2

Analía Rezk

Based on previous knowledge of our students, in the last course of Morphology the topic of spatial lines was approached from one of their possible generations: as a result of the intersection of spatial surfaces. From this point of view we explored relevant elements that are fundamental keys to turn these lines into powerful morphogenerative factors.

This is closely related to one of the aims of the third course of Morphology for Industrial Design, which is to qualify the students in the recognition of complex surface intersections as a strategy that produces new interpretations of well known forms, and to reveal the strong connection that the generative systems have with the forms involved in the intersections.

As the first step in the workshop, students built large scale 3D models of intersections between surfaces, as can be seen in Figure 1.

Figure 1. Model produced in the first meeting, based on the femisphere.

The purpose of this exercise was to visualize and to manipulate an intersection between spatial surfaces that later the students were going to solve in drawings. So, the topic was approached directly from the practice and from the previously acquired knowledge. The students pondered on the subject through the direct visualization of the problem using conceptual tools too. In order to be able to visualize and understand the determination of the line of intersection, students had to understand not only the abstract structure and the generative methods of the shapes involved, but their relevant sections and their morphological attributes as well.

The specific aim of the first stage was to instruct the student in the practice of finding spatial lines generated by the intersection of two given figures by means of drawing systems. They employed both, handmade sketches and digital tools, as is shown in Figure 2. In the exercise, students defined the limits of the overlapping area of the intersection and established cutting planes to determine the points of intersection between the sections of both figures.

This work focused in the conceptual understanding of the generative systems of spatial surfaces in general and, in particular, of those which take part in the analytical practice. The spatial relations between the operands of the intersection were thoroughly analyzed as a way of creating and rigorously transforming spatial lines.

Figure 2. Handmade and instrumental drawing of the graphic solution of a fragment of the intersection modeled on the first class. Student: Cortés Kenny

After this analytical practice, the following step, was the creation of continuous unions between spatial surfaces. This design resource is present in many everyday objects and is frequently associated with specific technologies such as plastic blowing process, injection, vacuum forming and those which work with flow of melted material:

metal casting, injection. This tool is also part of the communicational language of objects, which can be intentionally used. For all these reasons it is a relevant topic of the third course of Morphology.

Fillet and blend surfaces, which begun with digital modeling, create areas of transition and union between the intersected figures. They substitute the line of intersection by a spatial surface that exhibits or hides the Boolean operations according to its proportions and the variability of its generative lines. So, the emergent surfaces of the intersection become connected, at least with tangent continuity.

Figure 3. Rendering of minimum, medium and maximum fillets. Students: Ferrin, Patron Costas, Umansky, San Juan, Garcia Truchero

In the next assignment, the knowledge previously acquired in the preceding exercises was used to design new shapes, by means of intersections. The fundamental aim was to get the students to understand the possibility of creating spatial lines as a result of Boolean operations. Later, the controlled transformations produced adjustments in the spatial lines, by intentionally modifying the operators of the intersection. This allowed the students to design new surfaces, which combined attributes of those that originated them.

This practice had various stages which regulated the complexity of the problems to solve. In stage 1, shown in Figure 4, students had to create spatial lines through the intersection of known spatial surfaces. It was required that the spatial line should have isometric symmetry. So, a topic previously developed, was reviewed and extended. This practice also involved the knowledge of generation of surfaces and of their structure. Also, it allowed them to work with sketches as a preliminary approach, which was later defined with digital media.

Figure 4. Spatial line obtained by the intersection of a cone and a torus. Students: Magneto, Uthurriaga, Marini

Within the established requirements, students explored the generation of new surfaces through known shapes. So, new possibilities opened that they were able to control and handle intentionally through the conceptualization of the abstract structure of each shape and of the new combination.

In the second stage they produced a deep analysis of the previously obtained shapes and the control of their attributes. Students chose a volume emergent of the intersection of stage 1. They discovered that they could extract different shapes out of the same intersection, through addition, subtraction, the proper intersection or their combination. So, diverse interpretations were established based in different points of interest, as is shown in Figure 5. It also allowed them to see convex shapes as imprints on others, and so, as concave figures.

Figure 5. Intersection diagram of a hyperboloid of one sheet and a paraboloid of revolution, and the different shapes emergent from the operation. Students: Hoban, Cordero, Gardelli

Afterwards, they made a transformation on the selected shape, through homeometric operations in the Boolean operands or on the shape emergent of the operation. (Figure 6) This let the students deepen their knowledge on degrees of symmetry and on interpretations of form. Also it allowed a reflection on the direct influence of spatial relations between operands and of the modification of proportions of the figures that determine the spatial lines. This made the control of transformations easier and the intentional operation of forms.

Figure 6. Intersection diagram of a torus and a sphere, selection of an area that is homeometrically transformed. Student: Gechuvind

Later, in the next stage, shown in Figure 7, fillet surfaces were included as a design tool. Up till then, it was only an operative instrument. In order to change the way of using them, the assignment required that the students should highlight the transformation previously applied using this surfaces, forcing the limit of recognition of the identity of the former shape. The use of these surfaces on edges should be selective and a clear projecting resource with defined objectives. In this case, it was used to emphasize the homeometry of the form. The integration of the shapes that composed the new shape was regulated through variations of diameter of the fillets.

Figure 7. Use of fillet surfaces as transformation instrument of a design project. Student: Cortés Kenny

It was verified how the different generative systems and its lines were the elements of control of the new lines that surged of the intersection. It was demonstrated that, linking them to existing and known shapes, they turned to be mighty tools for the transformation of both, the operands of the intersections and the new shapes that resulted from them. Intersections became instruments of generation and comprehension of the attributes of spatial lines.

A fundamental factor in the relation between lines and the figures that determine them is that we can define the formers by the attributes of the figures involved, as they are intimately related.

Figure 8. 3D models of three steps in the practice.

**Conclusions:**

This series of practices aimed to let the students reach the theoretical conceptualization of spatial lines, of their attributes and generative possibilities starting with known surfaces. The knowledge of generative systems of spatial surfaces provided the means to make an intentional and deliberate use of the features of the lines and new shapes that resulted from the intersection. So, it opened a broad range of tools for the projecting practice and it enriched the instruments available for the Industrial Design student.

Considering that an Industrial Designer should count with the capacity to create new shapes the requirements and conditions for the guided experimentation were put forward in order to develop these resources. The objective was that the future professionals integrate their previous knowledge, build their own projecting tools and discover the broad field in which this area of design unfolds. There was a special highlight in the strengthening of the student’s capabilities of exploration, control and handling of abstract concepts in the study of form, and of its application and definition in the concrete level. Both instances are fundamental in the education of future Industrial Designers.

## Derivations and conclusions

Patricia Muñoz

When we decided to transfer the findings of the research to our classrooms we had to revise them in order to connect them to the contents and the practices of the three courses of Morphology. Throughout the experience we were able to verify the validity of the conceptual developments by checking them in the explorations of more than three hundred students.

Frequently, production is precedent to theory and, in this opportunity, professional work provided us with the first elements of our research and made evident the relevance and pertinence of this subject matter in the formative process of future industrial designers. So, professional practice, teaching and research came together and established relations of feedback which strongly motivated our students.

Even if the conceptual scaffolding enabled our students to make handmade sketches in the classroom, the use of CAD systems to adjust the first conceptual explorations was fundamental, particularly in the second and third course. However, in the three instances, the initial research was handmade.

The results of the workshop in the three levels was retrieved to the students through the Website, where they could see in detail their partners´ production and read comments which explained the value of the exhibited work. In the final lecture, which closed this experience for the three courses, the learning process was analyzed, narrating it by means of the work done throughout the experience. So, each student could acknowledge its contribution to the line of investigation developed in his course and also related to a broader context that included the other two. As a student said, the joint work allowed him to "see different ways of solving the assignments with different knowledge".

Figure 1. Students´ work of the three courses of Morphology in Industrial Design undergraduate program, 2007

Until 2007, when we carried out this experience, we had not included the subject of Spatial Lines as learning objective while teaching Morphology in Industrial Design, because we did not have a strong theoretical basis to go beyond playful explorations and produce significative learning. The contents of the research project enabled this experience which was rich and nourishing.

Even if we have not repeated the topic of spatial lines –because since we started this instructional strategy we have not repeated any of them- these shapes are still present in our assignments because they have become an available content that appears once and again inside other didactical issues we have developed since that time. Fortunately, we can say that spatial lines are today an everyday content of our courses but that they still surprise us endlessly with their beauty and morphogenerative possibilities.

## EPILOGUE

So, we have reached the end of what we wanted to tell you. Starting in the initial coincidence of the unexpected encounter with the past of Archytas, to our present projecting practices and to the instruction of future industrial designers; connecting these three moments with the same topic.

As any limit, is always an end and a beginning. However we understand that it is a closing point, not an end. It allows us to settle, at least temporarily, the answers we have found throughout this research and its transference to our students. We know these conclusions are not definite and reassuring. We do not believe this would be desirable either.

We hope to have aroused someone's curiosity, so they will venture on these explorations. Starting from shapes that are supposed to be well known but that pleasingly surprise us. That are unrelenting if we can inquire them form a restless, questioning and perhaps a little bit irreverent point of view. If we agree to look for what is not passively offered but for what requires a searching activity. These are shapes that permanently defy our knowledge about them; that invite us to unveil that which we still cannot discover, even if we can suspect it.

Because of all these reasons we can only finish repeating that this is only a closing point, provisional and necessary, but that it cannot be an end.

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